9-1 day 2 The Integral and p-test

Learning Objectives:

I can use the integral test to determine whether an infinite series converges or diverges

I can use the p-test to determine whether an infinite p-series converges or diverges

I can identify and understand the properties of the Harmonic Series.

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The Integral Test

If f(x) is positive, continuous, and decreasing for $x \geq 1$ and $f(n)=a_n$ for integers $n \geq 1$, then

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x)dx$$

Either BOTH converge or BOTH diverge.

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Ex1. Use the integral test to determine if each series converges or diverges.

$\lim_{n\to\infty} \frac{n}{n^2+1} = 0$

diverges

1.) $\sum_{n=1}^{\infty} \frac{n}{n^2+1} = \frac{1}{2} + \frac{2}{5} + \frac{3}{10} + \frac{4}{17} + \frac{5}{26} + \cdots$

$\int_1^{\infty} \frac{x}{x^2+1}\,dx = \lim_{B\to\infty} \int_1^{B} \frac{x}{x^2+1}\,dx$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$\int_1^b \frac{x}{2x\,u}\,du$

$\frac{1}{2}\int_1^b \frac{1}{u}\,du$

$\lim_{b\to\infty} \frac{1}{2}\ln x^2+1 \Big|_1^b$

$\frac{1}{2}\ln b^2+1 - \frac{1}{2}\ln 2 \quad \frac{1}{2}\ln \frac{b^2+1}{2}$

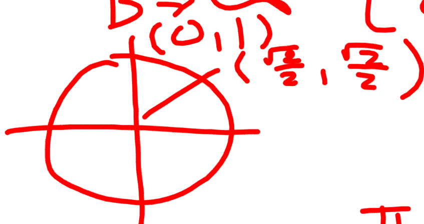
$= \infty$ diverges

2.) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converge

$\lim_{B\to\infty} \int_1^B \frac{1}{x^2+1}\,dx$

$\lim_{B\to\infty} \tan^{-1} x \Big|_1^B$

$\lim_{B\to\infty} \tan^{-1} B - \tan^{-1}(1)$



$\frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$ converges

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P-series

A p-series is a series of the form $\sum_{n=1}^{\infty} \frac{1}{n^p}$

p-test

A series $$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

1.) Converges if p > 1

2. Diverges if P $\leq$ 1

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$$\int_1^\infty \frac{1}{x^p}dx$$

From what we know about improper integrals, this integral will converge if $p > 1$ and diverge if $p \leq 1$. Hence, the p-test is really just an extension of the integral test.

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If p =1, $\sum_{n=1}^{\infty} \frac{1}{n}$, is called the Harmonic Series.

$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$

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Ex2. Determine whether each series converges or diverges.

1.) $\sum_{n=1}^{\infty} \frac{1}{n^{2/3}}$

$2/3 < 1$
diverges.

2.) $\sum_{n=1}^{\infty} \frac{1}{n^{5/3}}$

$5/3 > 1$
converge

3.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

$p = 1/2 < 1$
diverges

4.) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^4}}$

$p = 4/3 > 1$
converge

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Homework

Integral and p-test worksheet